J. Kontinen
J. Väänänen
D. Westerståhl

Editorial Introduction

Suppose a table of data concerning three variables x, y and z is given, for example:

x	y	z
1	0	1
5	1	5
3	1	1
1	0	5

(I)

If we look at the values of x and y we may observe that when x is the same also y is the same. The converse is not true: y can be the same on a row without x being the same. In the light of this data we say that y *functionally depends* on x but not conversely. If the concept "functionally depends" is imported into first order logic, *dependence logic* [5] emerges. This special issue contains nine research papers investigating different aspects of dependence logic and its predecessor, *independence friendly logic* [2].

We add the new atomic formula

$$=\!(x, y) \tag{II}$$

to first order logic with the meaning that y functionally depends on x. We think of the rows of the table (I) as assignments that assign values to variables. Thus we have a background model and the variables are meant to range over the elements of the model. Such tables of assignments are called *teams* in [5]. A team X satisfies (II) if

$$\forall s, s' \in X(s(x) = s'(x) \to s(y) = s'(y)).$$

The concept of a team satisfying a formula extends to all of dependence logic in a canonical way. The traditional concept of a single assignment s satisfying a first order formula corresponds to the singleton team $\{s\}$ satisfying the formula in the above sense, so dependence logic is a conservative extension of first order logic.

Special issue: Dependence and Independence in Logic
Edited by **Juha Kontinen, Jouko Väänänen, Dag Westerståhl**

Partially ordered quantification [1] can be expressed compositionally in dependence logic as follows:

$$\begin{pmatrix} \forall x & \exists y \\ \forall u & \exists v \end{pmatrix} \phi \leftrightarrow \forall x \exists y \forall u \exists v (=\!(u, v) \wedge \phi).$$

Independence friendly logic ([2]) extends first order logic by quantifiers of the form $\exists y/x$ with the intuitive meaning "there is a y independently of x". The semantics was originally game theoretic but can be also given in terms of teams ([3]) as follows: A team X satisfies $\exists y/x\phi$ if there is a team Y, obtained from X by adding a column for y (or modifying the y-column if it already exists) such that Y satisfies ϕ, the teams X and Y agree about variables other than y, and

$$\forall s, s' \in Y([\bigwedge_z s(z) = s'(z)] \rightarrow s(y) = s'(y)), \tag{III}$$

where z runs through relevant variables other than x. A simpler version, *dependence friendly logic*, obtains if instead of quantifiers $\exists y/x$ we add quantifiers $\exists y \backslash x$ with the meaning: A team X satisfies $\exists y \backslash x\phi$ if the above holds with (III) replaced by

$$\forall s, s' \in Y(s(x) = s'(x) \rightarrow s(y) = s'(y)).$$

In other words,

$$\exists y \backslash x\phi \leftrightarrow \exists y(=\!(x, y) \wedge \phi).$$

As is the case with partially ordered quantification, the expressive power of sentences of dependence logic and (in)dependence friendly logic is exactly Σ^1_1 i.e. existential second order logic. Since the semantics of dependence logic is defined via teams, we cannot reduce the semantics of formulas to the semantics of sentences obtained from the formulas by substituting constant symbols for free variables. So there is the new question, what the expressive power of formulas of dependence logic is. It turns out that if we use a new predicate symbol to refer to the team, the expressive power of formulas of dependence logic is exactly existential second order logic with the new predicate for the team occurring only negatively [4].

Dependence can be added also to other logics than first order logic. In propositional logic we can consider tables like

p_0	p_1	p_2
1	0	1
1	0	0
0	1	1

and observe that p_1 functionally depends on p_0, but p_2 does not. We can add the atoms

$$=(p_0, p_1)$$

(and more general similar atoms) to propositional logic and define truth with respect to a set X of valuations by saying that X satisfies $=(p_0, p_1)$ if

$$\forall v, v' \in X(v(p_0) = v'(p_0) \to v(p_1) = v'(p_1)).$$

There is a canonical way to extend this to modal logic, leading to a *modal dependence logic*. Other systems where dependence has led to interesting developments, recorded in the papers of this issue, are logic without identity, quantifier-free logic, intuitionistic logic, epistemic logic, probabilistic logic, and formal semantics.

The above discussion makes perfect sense in finite models leading to the observation that dependence logic gives a new language for NP, non-deterministic polynomial time. This observation has led to complexity theoretic investigations, which are largely still under way.

The "independence" in independence friendly logic is hidden in the clause "other than x" in (III). So this is independence by means of functional dependence on *other*. Recently a stronger form of independence was introduced. This new form is closely related to the concept of independence of random variables, but also to concepts of outcome-independence and parameter-independence in quantum physics. We include in this issue two contributions on this topic.

The goal of the study of dependence and independence in logic is to establish a basic theory of dependence and independence phenomena underlying seemingly unrelated subjects such as game theory, random variables, database theory, scientific experiments, and probably many others. The monograph [5] stimulated an avalanche of new results which have demonstrated remarkable convergence in this area. The concepts of (in)dependence in the different fields of science have surprising similarity and a common logic is starting to emerge. This special issue will give an overview of the state of the art of this new field.

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Juha Kontinen
Department of Mathematics and Statistics
University of Helsinki
Finland
juha.kontinen@helsinki.fi

Jouko Väänänen
Department of Mathematics and Statistics
University of Helsinki
Finland
and
Institute for Language, Logic and Computation
University of Amsterdam
The Netherlands
jouko.vaananen@helsinki.fi

Dag Westerståhl
Department of Philosophy
University of Stockholm
Sweden
dag.westerstahl@philosophy.su.se