Logic, games and quantifiers

Course description

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Introduction

This 7.5 hp course is mainly intended for students in philosophy and mathematics, and is generally accessible to an audience with some background on classical first-order logic and modal logic.

Practical information

The course will be given in English. It will comprise 13 double lecture sessions, given twice a week, on **Mondays and Wednesdays 13.00-16.00** (incl. breaks and discussion and exercise time), starting on **April 11, 2016 at 13.00 in room D734, Södra huset, hus D**. The course is planned to end on May 30.

Lecturers info:
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Brief description

Games play important role in defining, understanding and computing logical semantics. They can be used to evaluate the truth of a logical formula in a given model, to establish logical equivalence or non-equivalence of two models of a given logical language, or to show non-definability of a property of models in a given logical language. Generalised quantifiers feature very essentially in both natural language and formal logic. Their study uses quite naturally specific evaluation games and model comparison games.

This course will introduce, discuss and illustrate with examples some of the most important logical games, such as dialogue and argumentation games, formula evaluation (aka, model checking) games and model comparison games. It will also present a selection of the most interesting and natural generalised quantifiers, characterising their expressiveness and model-theoretic properties, using inter alia logical games.

Eventually we will discuss game-theoretic semantics of logical languages and some recently developed logics essentially defined using game-theoretic semantics.

Tentative course schedule:

1. **Week 15 (April 11-13), Lectures 1, 2 (VG)**
   (a) Logical games and applications: introduction.
   Obligatio games. Argumentation games. Lorenzen dialogue games.
   (b) Evaluation (model-checking) games for modal logic (ML) and first-order logic (FOL). Truth as existence of a winning strategy for the Verifier.
2. **Week 16 (April 18-20), Lectures 3, 4 (VG)**
   (a) Model comparison games. Game characterisation of logical equivalence.
   Bisimulations and bisimulation games for modal logic. Bisimulation invariance.

(b) Characteristic modal formulae. Bisimulation equivalence and infinitary modal equivalence.

3. **Week 18 (May 2-4), Lectures 5, 6 (VG)**

   (a) Model comparison games for FOL: Ehrenfeucht-Fraïsśe (EF) games. Game characterisation of elementary equivalence.

   (b) Partial and finite isomorphisms. Fraïsśe theorem.

   (c) Characteristic FO formulae. EF invariance and FO definability.

4. **Week 19 (May 9-11), Lectures 7, 8 (VG)**

   (a) Pebble games and bounded-variable fragments of FOL.

   (b) Using model comparison games to prove non-equivalence and non-definability results.

5. **Week 20 (May 16), Lecture 9 (VG)**

   (a) Model-building games.

   (b) Game theoretic semantics in general. Independence-friendly logic.

6. **Week 20 (May 18) Lecture 10 (DW)**

   Introduction to generalized quantifiers. Some history. Examples from natural language, and from logic and mathematics. Monadic and polyadic quantifiers. Monotonicity and other properties of quantifiers.

7. **Week 21 (May 23-25) Lectures 11, 12 (DW)**

   (a) Logics with generalized quantifiers: syntax and semantics. Properties of logics: compactness, axiomatizability, Löwenheim-Skolem properties. Definability. Comparing the expressive power of logics. Lindström's theorem.

   (b) Model comparison games for quantifiers 1: The expressive power of monadic quantifiers, such as most, an even number of, '(in)finitely many, as many as, more than, the Henkin quantifier. Facts about definability in terms of monotone quantifiers.

8. **Week 22 (May 30) Lecture 13 (DW)**

   Model comparison games for quantifiers 2: Applications to some polyadic quantifiers, such as connectivity (of graphs), branching quantifiers, Ramsey quantifiers, resumptive (vectorized) quantifiers, least fixed point quantifiers. The bijective game and non-definability from monadic quantifiers.

Course literature

The course literature will consist of a selection of readings from chapters of books and handbooks and some papers. All these are available online or will be provided electronically or otherwise. In addition, some summary slides will be provided after each lecture. Here are some major references:

1. Johan van Benthem, **Logic in Games**, MIT Press, 2014.
2. Stanley Peters and Dag Westerståhl, **Quantifiers in Language and Logic**, Clarendon Press, Oxford, 2006.
3. Jouko Väänänen, **Models and Games**, Cambridge University Press, 2011.
4. Jaakko Hintikka and Gabriel Sandu, "Game-theoretical semantics", chapter in J. van Benthem and A. ter Meulen (eds.), **Handbook of Logic and Language**, Elsevier, 1997, pp. 361–410.

Prerequisites

Background on classical first-order logic, comparable to the content of the course Metalogik 1.
Background on modal logic, including basics of possible worlds semantics.

Assessment

3 written assignments, to be posted in weeks 16, 19, 21.