Decomposing Generalized Quantifiers*

Dag Westerståhl
Department of Philosophy
University of Gothenburg

Abstract

This note explains the circumstances under which a type $\langle 1 \rangle$ quantifier can be *decomposed* into a type $\langle 1, 1 \rangle$ quantifier and a set, by fixing the first argument of the former to the latter. The motivation comes from the semantics of noun phrases (also called determiner phrases) in natural languages, but here I focus on the logical facts. However, my examples are taken among quantifiers appearing in natural languages, and at the end I sketch two more principled linguistic applications.

1 Introduction

The motivation for the results in this note comes from linguistics: Determiners and other phrases with a similar function in natural languages denote type $\langle 1, 1 \rangle$ (generalized) quantifiers, and noun phrases (NPs or DPs) denote type $\langle 1 \rangle$ quantifiers, often but not always obtained by restricting or *freezing* the noun argument of a $\langle 1, 1 \rangle$ quantifier. I study the 'inverse' of freezing: *decomposition* of a type $\langle 1 \rangle$ quantifier (when possible) by means of a set and a type $\langle 1, 1 \rangle$ quantifier.

Here I focus on the logical facts: characterizations of classes of decomposable quantifiers are provided, and questions of uniqueness of the decomposition are addressed. The results are not difficult, but they yield a fairly clear picture of the situation, and can be directly applied to certain issues in semantics. Two such applications – to reciprocals and to possessives – are sketched in the final section; the latter are dealt with extensively in Peters and Westerståhl [5].

Section 2 gives background on (generalized) quantifiers and their properties, section 3 provides the relevant notions of freezing and decomposition, section 4 presents and proves the results, and section 5 presents linguistic applications. Some observations on decomposition and freezing were made in Peters

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and Westerståhl [4] (e.g. Ch. 4.5.5); here they are generalized and put in a coherent picture.

2 Background on quantifiers

To make the presentation more self-contained, central notions and definitions are provided in this section. Notation is as in [4], where (much) more background information on quantifiers in language and logic can be found.

2.1 Quantifiers of type $\langle 1, 1\rangle$ and $\langle 1\rangle$

Except for one example in section 5, we only deal with quantifiers of these two types.

> **Definition**
>
> A (*generalized*) *quantifier of type* $\langle 1, 1\rangle$ Q associates with each universe M a binary relation Q_M between subsets of M. In the type $\langle 1\rangle$ case, Q_M is a unary relation. Logicians are more used to see Q a *class of structures*. This is essentially just a notational difference; in the type $\langle 1, 1\rangle$ case:
>
> $$(M, A, B) \in Q \iff Q_M(A, B)$$

Type $\langle 1, 1\rangle$ quantifiers interpreting English determiner phrases are conveniently named by those phrases, as in the examples below. For all M and all $A, B \subseteq M$,

$$\begin{aligned}
&\textit{all}_M(A, B) \iff A \subseteq B \\
&(\textit{all}_{\text{ei}})_M(A, B) \iff \emptyset \neq A \subseteq B \\
&\textit{no}_M(A, B) \iff A \cap B = \emptyset \\
&\textit{at least two}_M(A, B) \iff |A \cap B| \geq 2 \\
&\textit{exactly five}_M(A, B) \iff |A \cap B| = 5 \\
&\textit{all but three}_M(A, B) \iff |A - B| = 3 \\
&\textit{more than two-thirds of the}_M(A, B) \iff |A \cap B| > 2/3 \cdot |A| \\
&\textit{most}_M(A, B) \iff |A \cap B| > |A - B| \\
&\textit{the ten}_M(A, B) \iff |A| = 10 \text{ and } A \subseteq B \\
&\textit{John's}_M(A, B) \iff \emptyset \neq A \cap \{a : \text{John `possesses' } a\} \subseteq B \\
&\textit{no except John}_M(A, B) \iff A \cap B = \{j\} \\
&\textit{infinitely many}_M(A, B) \iff A \cap B \text{ is infinite} \\
&\textit{an even number of}_M(A, B) \iff |A \cap B| \text{ is even}
\end{aligned}$$

$\textit{all}_{\text{ei}}$ is 'all with existential import': it is the quantifier that Aristotle (as well as many modern linguists, but few modern logicians) took "every" to mean. Of course, lots of type $\langle 1, 1\rangle$ quantifiers are much more remotely related to natural language:

$$sqrt_M(A,B) \iff |A \cap B| > \sqrt{|A|}$$
$$I_M(A,B) \iff |A| = |B| \quad \text{(the Härtig quantifier)}$$

As to type $\langle 1 \rangle$ quantifiers, all but the last two of the following are familiar from logic:

$$\forall_M(B) \iff B = M$$
$$\exists_M(B) \iff B \neq \emptyset$$
$$(\exists_{=n})_M(B) \iff |B| = n$$
$$(I_a)_M(B) \iff a \in B \qquad \text{(a Montagovian individual)}$$
$$Q^R_M(B) \iff |B| > |M - B| \quad \text{(the Rescher quantifier)}$$
$$(Q_0)_M(B) \iff B \text{ is infinite}$$
$$(C^{pl,u})_M(B) \iff \emptyset \neq C \subseteq B$$
$$(C^{pl,e})_M(B) \iff C \cap B \neq \emptyset$$

For each M, $(I_a)_M$ is the principal filter on $P(M)$ generated by a; it is also what Montague used to interpret *proper names*, so that the sentences

(1) John smokes.

(2) Some students smoke.

are interpreted on the same principle, as

$$[\![\mathrm{NP}]\!]_M([\![\mathrm{smoke}]\!])$$

where M is the discourse universe, and $[\![\mathrm{NP}]\!] = some^{[\![\mathrm{student}]\!]}$ (defined in section 3 below) in (2), and $= I_j$ in (1).

The quantifiers $C^{pl,u}$ and $C^{pl,e}$ can be used to obtain the universal and existential interpretation of *bare plurals* along the same lines, as in, respectively,

(3) Firemen are brave.

(4) Firemen are available.

It should be emphasized that quantifiers are *global*: with each universe M they associate a second-order relation (a local quantifier) over M. This is faithful to the meaning of the corresponding phrases in natural languages. To know what "some" means is not to know different things for different universes of discourse; it is to know just one thing, namely, the quantifier *some*.[1] In the case of Montagovian individuals, this means that if $j \notin M$, $(I_j)_M(B)$ is always false, which seems perfectly reasonable.

[1] In the present context, "meaning" stands for extension; intensions, possible worlds, etc. will not be relevant.

2.2 Basic properties of quantifiers

A crucial feature of quantifiers involved in natural language semantics, and in logic too for that matter, is that they *mean the same thing on every universe.* The following property comes very close to capturing this intuitive idea.

Definition

Q satisfies *extension* (EXT) iff whenever $A, B \subseteq M \subseteq M'$, we have $Q_M(A, B) \Leftrightarrow Q_{M'}(A, B)$. Similarly for the type $\langle 1 \rangle$ case; EXT makes sense for quantifiers of any type.

Thus, extending the universe (but not the arguments of the quantifier) has no effect. It is easy to define quantifiers that mean different things on different universes (e.g. one meaning *some* on universes with fewer than 10 elements, and *all* on larger universes), but these don't appear in natural language semantics. All type $\langle 1, 1 \rangle$ quantifiers above, and all type $\langle 1 \rangle$ quantifiers except $\forall$ and Q^R, are EXT. In general, it seems that all type $\langle 1 \rangle$ natural language quantifiers are EXT, except some of those that explicitly refer to the universe, e.g. by means of the predicate *thing*: *John, firemen, three cats, all but five students, some things, at most three things* are all EXT, whereas *everything, most things* are not.

For EXT quantifiers, mention of the universe M can be omitted; this is done whenever possible below.

Definition

Q is *closed under isomorphism* (ISOM) iff the corresponding class of structures is closed under isomorphism. In the type $\langle 1, 1 \rangle$ case, this is equivalent to the requirement that $Q_M(A, B)$ only depends on the *cardinals* of $A-B$, $A \cap B$, $B - A$, $M - (A \cup B)$.

Many but not all natural language quantifiers are ISOM; the exceptions above are those explicitly mentioning some individual or property, like *Mary, all except John, every student's*, etc. For logicians it is natural to treat these individuals or properties as arguments too. For example, one could think of the quantifier at work in *every student's* as having type $\langle 1, 1, 1 \rangle$: *every C's A is B*. Similarly, one could think of *Mary's* as taking two sets and one individual as arguments. These quantifiers – we could call them *fully abstracted* – would be ISOM, but the problem is that they have the wrong category. *Every student's* and *Mary's* are in important ways similar to *some* or *most*: they take a noun argument to form an NP, just like other determiners, so from a linguistic point of view, they should have type $\langle 1, 1 \rangle$. Similarly, NPs formed by a determiner and a noun should have type $\langle 1 \rangle$, not type $\langle 1, 1 \rangle$.

The natural way to deal with this issue is to have a *background model* $\mathcal{M}_0$ fix the interpretation of the relevant proper names or nouns. Provided the fully abstracted quantifiers are EXT, there is a *unique* way to treat expressions like *every student's, Mary's, John, three cats,* etc. as quantifiers of the desired types ($\langle 1, 1 \rangle$ or $\langle 1 \rangle$), relative to $\mathcal{M}_0$ (see Peters and Westerståhl [4], Ch. 3.5). Naturally, however, ISOM is lost.

Isom and Ext apply to arbitrary quantifiers. The next property, on the other hand, concerns only quantifiers having a *restriction argument* of type $\langle 1 \rangle$, in particular, those of type $\langle 1, 1 \rangle$.

Definition

A type $\langle 1, 1 \rangle$ quantifier Q is *conservative* (Conserv) iff for all M and all $A, B \subseteq M$, $Q_M(A, B) \Leftrightarrow Q_M(A, A \cap B)$.

All determiner interpretations are Conserv and Ext. This fact is responsible for much of the special behavior of natural language quantification, and in particular for all the results in the present note. For such quantifiers, $Q_M(A, B)$ depends only on the sets $A - B$ and $A \cap B$. If Q in addition is Isom, only the cardinality of those two sets matter.

Boolean operations on quantifiers (of the same type) are defined in the obvious way. In addition to ordinary negation, sometimes called *outer negation*, there is also an *inner negation* for quantifiers of these types; define, in the type $\langle 1 \rangle$ case, the quantifier $Q\neg$ by

$$(Q\neg)_M(B) \iff Q_M(M - B)$$

In the type $\langle 1, 1 \rangle$ case, we replace the second argument by its complement.

Finally, we shall need suitable notions of *triviality*:

Definition

A local quantifier Q_M is *trivial* iff it holds either of all subsets of M or of no such subsets. Q is *trivial* iff each Q_M is trivial.

Below, we sometimes restrict attention to *finite* universes; this is indicated by writing Fin.

3 Freezing and decomposition

3.1 Freezing

The first argument of a type $\langle 1, 1 \rangle$ quantifier denoted by a determiner is called the *noun* or *restriction* argument, and the idea of freezing is simply to fix that argument to some given set C. However, the result should be a global type $\langle 1 \rangle$ quantifier, defined on every universe M, whether M contains C or not. This is done as follows.

Definition

If Q is any type $\langle 1, 1 \rangle$ quantifier and C any set, define the type $\langle 1 \rangle$ quantifier Q^C, the *freezing* of Q to C, by

$$(5)\quad (Q^C)_M(B) \iff Q_{M \cup C}(C, B)$$

for every M and every $B \subseteq M$.

Thus, the universe is expanded, if necessary, so that it includes C. Though the idea of freezing is familiar, the definitions in the literature are often vague as to how to read Q^C on an arbitrary universe. One could, instead, keep M and cut down C to $C \cap M$, or one could simply stipulate that $(Q^C)_M(B)$ is false when $C \not\subseteq M$. Note that all three alternatives are equivalent when $C \subseteq M$. It is argued in Peters and Westerståhl [4] that (5) is the *correct* definition, mainly because it preserves Ext:

Fact 1
If Q is Ext*, so is Q^C.*

Proof. If $B \subseteq M \subseteq M'$: $(Q^C)_M(B) \Leftrightarrow Q_{C\cup M}(C,B) \Leftrightarrow Q_{C\cup M'}(C,B)$ (Ext) $\Leftrightarrow$ $(Q^C)_{M'}(B)$. □

The other definitions suggested above do not preserve Ext. In addition, it can be seen that the facts about freezing and decomposition to be established below, as well as a number of other similar results, depend on defining Q^C in the correct way, i.e. as in (5).

3.2 Decomposition

Definition

(i) A type $\langle 1\rangle$ quantifier Q is *decomposable* iff there is a set C and a Conserv and Ext type $\langle 1,1\rangle$ quantifier Q_1 such that $Q = Q_1^C$.

(ii) If Q_1 is also Isom, Q is said to be Isom *decomposable.*

Since Q_1 is required to be Conserv and Ext, decomposition is an inverse to freezing in a precise sense explained in Peters and Westerståhl [4] (Ch. 4, Fact 4). Note that without any requirements on Q_1, the notion of (Isom) decomposability would be empty:

Fact 2
For every type $\langle 1\rangle$ quantifier Q there is a set C and a type $\langle 1,1\rangle$ quantifier Q_1 such that $Q = Q_1^C$.

Proof. Define

$$(Q_1)_M(A,B) \iff A = \emptyset \ \& \ Q_M(B)$$

Then $(Q_1)^{\emptyset}_M(B) \Leftrightarrow (Q_1)_M(\emptyset, B) \Leftrightarrow Q_M(B)$, so $Q = (Q_1)^{\emptyset}$. Note that Q_1 is not Conserv, but it is Ext (respectively Isom) if Q is Ext (Isom). □

Here are some familiar examples of Isom decomposable quantifiers:

(6) $I_a = all^{\{a\}} = (all_{\mathrm{ei}})^{\{a\}} = (the_{\mathrm{sg}})^{\{a\}} = some^{\{a\}}$

(7) $I_a \wedge I_b = all^{\{a,b\}} = (all_{\mathrm{ei}})^{\{a,b\}} = (the_{\mathrm{pl}})^{\{a,b\}}$ $(a \neq b)$

(8) $I_a \vee I_b = \mathit{some}^{\{a,b\}}$

(9) $C^{pl,u} = (\mathit{all}_{\mathrm{ei}})^C$

(10) $C^{pl,e} = \mathit{some}^C$

Examples of decomposable but not Isom decomposable quantifiers, and of Ext but not decomposable type $\langle 1 \rangle$ quantifiers, will be given below.

A decomposable quantifier is only 'active' on the set to which the underlying type $\langle 1, 1 \rangle$ quantifier is frozen. The following definition and result make this claim precise.

Definition

Let Q_1, Q_2 be Conserv and Ext type $\langle 1, 1 \rangle$ quantifiers.

(i) Q_1 and Q_2 *agree on* C iff for all sets B, $Q_1(C, B) \Leftrightarrow Q_2(C, B)$.

(ii) Q_1 is Isom *on* k (where k is any cardinal), iff for all M and $A, B \subseteq M$, and all M' and $A', B' \subseteq M'$, if $|A| = |A'| = k$, $|A \cap B| = |A' \cap B'|$, and $|A - B| = |A' - B'|$, then $(Q_1)_M(A, B) \Leftrightarrow (Q_1)_{M'}(A', B')$.

Fact 3

For Conserv *and* Ext Q_1 *and* Q_2*:*

(a) *If* $Q = Q_1^C$ *and* Q_1 *and* Q_2 *agree on* C*, then* $Q = Q_2^C$.

(b) *If* Q_1 *is* Isom*, then* Q_1 *is* Isom *on* k *for every* k.

(c) *If* $Q = Q_1^C$ *where* Q_1 *is* Isom *on* $|C|$*, then* Q *is* Isom *decomposable.*

Proof. (a) and (b) are immediate. As to (c), if Q, Q_1, and C are as assumed, define Q_3 by

$$(Q_3)_M(A, B) \iff \begin{cases} (Q_1)_M(A, B) & \text{if } |A| = |C| \\ A \cap B \neq \emptyset \text{ (say)} & \text{otherwise} \end{cases}$$

Clearly, Q_3 is Conserv and Ext. Distinguishing the cases $|A| = |C|$ and $|A| \neq |C|$, we see that Q_3 is also Isom (in the former case, since Q_1 is Isom on $|C|$, and in the latter case, since Q_3 then amounts to *some*). In addition, Q_1 and Q_3 agree on C, so it follows from (a) that $Q = Q_3^C$. □

Fact 3 has the following immediate

Corollary 4

Q is Isom *decomposable if and only if* $Q = Q_1^C$ *for some* C *and some* Conserv *and* Ext Q_1 *which is* Isom *on* $|C|$.

4 Results

4.1 Global quantifiers living on sets

We are looking for some property of type $\langle 1\rangle$ quantifiers which gives a necessary and sufficient condition for decomposability. By Fact 1, one necessary condition is Ext; $\forall$ and the Rescher quantifier (*everything* and *most things*) are not decomposable already because they are not Ext. It turns out that the right place to start is the *live-on* property introduced in Barwise and Cooper [2]. Conservativity is crucial to most facts about global quantifiers in natural language, but Barwise and Cooper, who were among the first to realize its importance, thought of it as a property of *local* quantifiers:

> **Definition**
>
> Let C be any set. A local type $\langle 1\rangle$ quantifier Q_M *lives on* C iff for all $B \subseteq M$, $Q_M(B) \Leftrightarrow Q_M(C \cap B)$.

The connection with conservativity (which is immediate if the right notion of freezing has been chosen) is the following:

Fact 5
A type $\langle 1, 1\rangle$ quantifier Q is Conserv *iff for each set C and each universe M, $(Q^C)_M$ lives on C.*

Here are some easily verified facts about the live-on property (cf. Peters and Westerståhl [4], ch. 3, Lemma 1):

Fact 6

(a) *Q_M always lives on M, but need not live on any proper subset of M.*

(b) *Q_M lives on $\emptyset$ iff it is trivial.*

(c) *If Q_M lives on C and on D, it lives on $C \cap D$.*

(d) *If Q_M lives on C and $C \subseteq D$, Q_M lives on D.*

We didn't require that $C \subseteq M$ in the definition of "Q_M lives on C", but the above fact shows that Q_M lives on C if and only if Q_M lives on $M \cap C$. Indeed, it shows that

$$\{X \subseteq M : Q_M \text{ lives on } X\}$$

is a filter on $P(M)$, which is proper iff Q_M is non-trivial. If M is finite, it follows that Q_M lives on

$$W_{Q_M} = \bigcap\{X \subseteq M : Q_M \text{ lives on } X\}$$

which then generates the filter.

We now extend the live-on notion to global quantifiers as follows:

Definition

Let C be a set. A (global) type $\langle 1\rangle$ quantifier Q *lives on* C iff for all M, Q_M lives on C.

Fact 6 (b)–(d) carry over directly to the extended notion:

Fact 7

(a) *Q lives on $\emptyset$ iff it is trivial.*

(b) *If Q lives on C and on D, it lives on $C \cap D$.*

(c) *If Q lives on C and $C \subseteq D$, Q lives on D.*

If Q lives on some set, we let

$$W_Q = \bigcap\{X : Q \text{ lives on } X\}$$

Fact 8

(FIN) *If Q lives on some set, it lives on W_Q. That is, W_Q is then the smallest set that Q lives on.*

Proof. We are assuming that Q lives on some finite set C. Then, clearly,

$$W_Q \;=\; \bigcap\{X \cap C : Q \text{ lives on } X\}$$

But on the right-hand side we have an intersection of finitely many sets. Thus, by Fact 7(b), Q lives on W_Q.[2] □

The next fact relates W_Q to W_{Q_M}.

Fact 9

(FIN) *If Q lives on some set then, for all M, $W_{Q_M} \subseteq W_Q$.*

Proof. It is enough to show that if Q lives on a set C, then for all M, $W_{Q_M} \subseteq C$. But Q_M lives on C by definition, and hence on $M \cap C$. And W_{Q_M} is the smallest subset of M that Q_M lives on, so $W_{Q_M} \subseteq M \cap C$. Thus, $W_{Q_M} \subseteq C$. □

[2] This fails without the restriction to finite sets. Let Q be *infinitely many numbers*, i.e. $Q = \textit{infinitely many}^N$, where $N = \{0, 1, 2, \ldots\}$. Q lives on N, but also on $N - \{0\}$, $N - \{0, 1\}$, ... So there is no smallest set on which it lives, and W_Q as defined above is empty.

Note that if Q lives on some set C, W_Q is well-defined: it is a subset of C defined by a first-order set-theoretic formula.

The notation W_Q was used in a slightly different sense in [4], ch. 4.6, namely, for certain Q such that W_{Q_M} is independent of M (when Q_M is non-trivial). But when this is the case, that notion coincides with the one defined here.

4.2 Decomposition and the live-on property

Even though the live-on property for global quantifiers draws upon Barwise and Cooper's local version, it is a very different property. The local version essentially codifies conservativity, and so is to be expected in all natural language contexts. Also, *every* local quantifier lives on some set (its universe). Global quantifiers, one the other hand, 'normally' don't live on any sets at all. For example, a non-trivial and Isom type $\langle 1 \rangle$ quantifier lives on no set (Fact 13 below). In fact, the ones that do live on some set are essentially the decomposable quantifiers, as we will now see.

The next lemma is immediate (either directly from definitions or from Facts 5 and 7(c)):

Lemma 10
If Q is Conserv*, Q^C lives on C and all its supersets.*

We obtain the following characterization.

Theorem 11
Q is decomposable if and only if it is Ext *and lives on some set.*

Proof. Suppose Q is decomposable as Q_1^C. Then Q is Ext, and by Lemma 10, Q lives on C.

In the other direction, suppose Q is Ext and lives on C. Define Q_1 by

$$(Q_1)_M(A, B) \iff A = C \;\&\; Q_M(A \cap B)$$

Q_1 is Conserv by definition, and Ext since Q is Ext. We have, for all M and all $B \subseteq M$,

$$\begin{aligned} (Q_1^C)_M(B) &\Leftrightarrow (Q_1)_{M \cup C}(C, B) \\ &\Leftrightarrow Q_{M \cup C}(C \cap B) \quad \text{(by definition)} \\ &\Leftrightarrow Q_M(C \cap B) \quad \text{(by Ext)} \\ &\Leftrightarrow Q_M(B) \quad \text{(since } Q_M \text{ lives on } C\text{)} \end{aligned}$$

That is, $Q = Q_1^C$. □

Corollary 12

(a) *If Q is decomposable with C as the underlying set, then for every $D \supseteq C$, Q is decomposable with D as the underlying set.*

(b) *The class of decomposable type $\langle 1 \rangle$ quantifiers is closed under Boolean operations, including inner negation and dual.*

Proof. (a) follows from the theorem and Lemma 10. As to (b), the claims about negations (and hence duals) follow from the easily verified observations that for Conserv and Ext Q_1,

(11) $\neg(Q_1^C) \;=\; (\neg Q_1)^C$

and

(12) $(Q_1^C)\neg \;=\; (Q_1\neg)^C$

Now suppose $Q = Q_1^{C_1}$ and $Q' = Q_2^{C_2}$, for CONSERV and EXT Q_1 and Q_2. By Lemma 10, both Q and Q' live on $C_1 \cup C_2$. But then it readily follows that $Q \wedge Q'$ and $Q \vee Q'$ also live on $C_1 \cup C_2$. Since they are also EXT, they are decomposable, by the theorem. □

We can use the easy direction of the characterization to show that various type $\langle 1\rangle$ quantifiers, including many denoted by noun phrases in natural languages, are *not* decomposable. To begin, *something* and *nothing* ($\exists$ and $\neg\exists$) are EXT but not decomposable. More generally, the quantifiers *at least n things* ($\exists_{\geq n}$), for $n \geq 1$, and all non-trivial Boolean combinations of these, including inner negations and duals, are not decomposable. These include cases like *everything* ($\forall$) and *all but three things* ($(\exists_{\geq 3} \wedge \exists_{\leq 3})\neg$), which are not even EXT, and so *a fortiori* not decomposable, but also EXT noun phrase denotations like *between three and five things* ($\exists_{\geq 3} \wedge \exists_{\leq 5}$), not to mention *an even number of things*, etc. All of this follows from the simple fact that these quantifiers are (non-trivial and) ISOM:

Fact 13
If a type $\langle 1\rangle$ quantifier is non-trivial and ISOM, *it is not decomposable.*

Proof. Suppose Q is non-trivial and ISOM. By Fact 1, we may also suppose that Q is EXT.

Case 1: $\neg Q(\emptyset)$. By non-triviality, there is some M and some $B \subseteq M$ such that $Q_M(B)$. By EXT, it follows that $Q_B(B)$. Now suppose that Q lives on some set C. Take B' such that $|B'| = |B|$ and $C \cap B' = \emptyset$. It follows, since Q is ISOM, that $Q_{B'}(B')$, and hence, since $Q_{B'}$ lives on C, that $Q_{B'}(C \cap B')$. But this contradicts the assumption of the case. Thus, Q doesn't live on any set, and is therefore not decomposable.

Case 2: $Q(\emptyset)$. Then the previous argument shows that $Q' = \neg Q$ is not decomposable. But then, by (11), neither is Q. □

What about EXT but not ISOM quantifiers? Among noun phrase denotations, we find the following. Define, for any set D, the quantifier *only D* by

(13) $(only\ D)_M(B) \iff \emptyset \neq B \subseteq D$.

These are at work in the interpretation of sentences like

(14) Only John left the party. ($D = \{j\}$)

(15) Only firemen wear helmets.

Fact 14
Quantifiers of the form only D *are* EXT*, but not decomposable when* $D \neq \emptyset$.

Proof. That EXT holds is clear from the definition of *only D*. Suppose *only D* were decomposable, and hence lives on some set C. Take any $a \notin C \cup D$. Since $(only\ D)(D)$ holds ($D \neq \emptyset$), we obtain $(only\ D)(C \cap D)$, and therefore $(only\ D)(C \cap (D \cup \{a\}))$, so $(only\ D)(D \cup \{a\})$, which contradicts (13). □

This underscores the familiar observation in linguistics that, first appearances notwithstanding, *only* is in fact not an English determiner (e.g. Peters and Westerståhl [4], p. 139, fn. 15). If it were a determiner, one would expect *only firemen* to be interpreted as a frozen type $\langle 1, 1\rangle$ quantifier, but Fact 14 says it cannot be.

4.3 Characterizing ISOM decomposability

We have seen several examples of ISOM decomposable quantifiers, as well as EXT but non-decomposable quantifiers, but what about quantifiers that are decomposable but not ISOM decomposable? A host of such quantifiers can be identified using the characterization of ISOM decomposability in this subsection.

> **Definition**
> A type $\langle 1\rangle$ quantifier Q *doesn't distinguish subsets of* C *of the same size* iff whenever $B_1, B_2 \subseteq C \cap M$ are such that $|B_1| = |B_2|$ and $|C - B_1| = |C - B_2|$, $Q_M(B_1) \Leftrightarrow Q_M(B_2)$.

Theorem 15
Q is ISOM *decomposable if and only if Q is* EXT *and lives on some set C such that Q doesn't distinguish subsets of C of the same size.*

Proof. If $Q = Q_1^C$ for some CONSERV, EXT, and ISOM Q_1, then Q lives on C, and it is immediate that Q doesn't distinguish subsets of C of the same size.

In the other direction, suppose Q is EXT and lives on a set C such that Q doesn't distinguish subsets of C of the same size. Define Q_1 by

$$(Q_1)_M(A,B) \Leftrightarrow |A| = |C| \ \&\ \exists X \subseteq C(Q_M(X) \ \&\ |X| = |A \cap B| \ \&\ |C - X| = |A - B|)$$

Q_1 is CONSERV by definition, and EXT since Q is EXT. Also, Q_1 is clearly ISOM on $|C|$. Moreover, we have, for any B,

$$\begin{aligned} Q_1(C, B) \quad &\Leftrightarrow \quad \exists X \subseteq C(Q_M(X) \ \&\ |X| = |C \cap B| \ \&\ |C - X| = |C - B|) \\ &\qquad \text{(by definition)} \\ &\Leftrightarrow \quad Q(C \cap B) \quad \text{(by assumption)} \\ &\Leftrightarrow \quad Q(B) \quad \text{(since } Q \text{ lives on } C\text{)} \end{aligned}$$

So $Q = Q_1^C$, and Q is ISOM decomposable by Fact 3(c). $\square$

Consider $Q = I_j \vee (I_m \wedge I_s)$, which interprets the noun phrase *John, or Mary and Sue*, as in

(16) John, or Mary and Sue, will come to the party.

The atoms are ISOM decomposable, as we have seen, and it follows from Corollary 12(b) that Q is decomposable. Suppose Q were ISOM decomposable as Q_1^D. We have $Q_1(D, \{j\})$, hence $Q_1(D, D \cap \{j\})$ by CONSERV, so $j \in D$, since $\neg Q_1(D, \emptyset)$. A similar argument shows that either $m \in D$ or $s \in D$. Thus, since $Q(\{j\})$ but neither $Q(\{m\})$ nor $Q(\{s\})$ holds, we have contradicted Theorem 15.[3]

Here is a generalization of this observation.

Fact 16
The quantifier some men or all women *is decomposable but not* ISOM *decomposable. More generally, the quantifier $Q = some^D \vee all^E$ is decomposable, but not* ISOM *decomposable, if $D \neq \emptyset$, $|E| > 1$, and $D \cap E = \emptyset$.*

Proof. (Note that $I_j \vee (I_m \wedge I_s)$ is the special case when $D = \{j\}$ and $E = \{m, s\}$.) The claim about decomposability follows from Corollary 12(b). Next, by our assumptions, we have for all $a \in D$, $Q(\{a\})$, whereas for all $b \in E$, $\neg Q(\{b\})$. Thus, by Theorem 15, it suffices to show that if $Q = Q_1^C$, we have $C \cap D \neq \emptyset$ and $C \cap E \neq \emptyset$. But this follows from the conservativity of Q_1: Since $Q_1(C, D)$, we have $Q_1(C, C \cap D)$, and hence $C \cap D \neq \emptyset$, since $\neg Q_1(C, \emptyset)$, and similarly for E. $\square$

4.4 Uniqueness

Suppose Q is (ISOM) decomposable. When is the decomposition unique, i.e. when can we recover the underlying type $\langle 1, 1\rangle$ quantifier and the underlying set? The first part of the question is easy. We already saw that in many cases the underlying quantifier is not unique: $I_a = all^{\{a\}} = (all_{\mathrm{ei}})^{\{a\}} = (the_{\mathrm{sg}})^{\{a\}} = some^{\{a\}}$, etc. In fact, it can *never* be recovered:

Fact 17
For every decomposable quantifier Q one can find Q_1, Q_2 and C such that $Q_1 \neq Q_2$ and $Q = Q_1^C = Q_2^C$.

Proof. This is more or less immediate from Fact 3: If $Q = Q_1^C$, just let Q_2 differ from Q_1 on some D with $|D| \neq |C|$, but be the same as Q_1 otherwise. $\square$

[3]But in this case the conclusion follows directly from the fact that Q_1 is ISOM: if $m \in D$, $Q_1(D, \{j\})$ implies $Q_1(D, \{m\})$ by ISOM.

The second part of the question is more interesting. From Corollary 12(a) we know that it is only in the case of Isom decomposability that we have any chance to recover the underlying set.

Lemma 18
(Fin) *Suppose Q is non-trivial and decomposable as Q_1^C, where Q_1 is* Isom *on $|C|$. Suppose further that Q lives on some set D. Then $C \subseteq D$.*

Proof. Suppose first that $\neg Q(\emptyset)$ holds. Let B_0 be a set of the *smallest size* such that $Q(B_0)$. (Since Q is Ext by assumption, we can leave out the universe as usual.) Such a set exists by non-triviality. By assumption, $B_0 \neq \emptyset$. Moreover, by the conservativity of Q_1 (or the fact that Q_M lives on C), we may assume that $B_0 \subseteq C$.

Suppose $a \in C - D$. Take $B \subseteq C$ such that $a \in B$ and $|B| = |B_0|$. Since Q_1 is Isom on $|C|$, C is finite, and $Q_1(C, B_0)$ holds, it follows that $Q_1(C, B)$, and thus $Q(B)$. Since Q lives on D we get $Q(D \cap B)$ and thus $Q(D \cap (B - \{a\}))$, since $a \notin D$. But then $Q(B - \{a\})$, again because Q lives on D. Since B is finite, this contradicts the assumption that it was of the smallest size such that $Q(B)$.

Thus, $C \subseteq D$. If instead $Q(\emptyset)$ holds, we apply the above reasoning to $\neg Q$, which also satisfies the assumptions in the lemma. □

Theorem 19
(Fin) *If Q is non-trivial and* Isom *decomposable, its underlying set is uniquely determined. More exactly, there is a unique set C such that for some Q_1 which is* Conserv, Ext, *and* Isom *(on $|C|$), $Q = Q_1^C$.*

Proof. Suppose $Q = Q_1^C = Q_2^D$, where Q_1, Q_2 are Conserv and Ext, and Q_1 is Isom on $|C|$, whereas Q_2 is Isom on $|D|$. By Lemma 10, Q lives on D. Thus, by Lemma 18, $C \subseteq D$. A symmetric argument shows that $D \subseteq C$, so $C = D$. The corresponding claim of uniqueness when Q_1, Q_2 are Isom follows via Fact 3(b). □

Let

$$Un(Q, Y) \iff Q = Q_1^Y \text{ for some Conserv, Ext, and Isom } Q_1$$

By the theorem, assuming Fin, the relation Un is single-valued for non-trivial Q, so we can define a function U from type $\langle 1 \rangle$ quantifiers to sets by

$$U(Q) = \begin{cases} Y & \text{if } Q \text{ is non-trivial and } Un(Q, Y) \\ \emptyset & \text{otherwise} \end{cases}$$

U, restricted to Isom decomposable quantifiers, can also be defined in terms of Q alone. Recall the definition of W_Q in section 4.1.

Corollary 20
(Fin) *If Q is* Isom *decomposable, then $U(Q) = W_Q$.*

Proof. If Q is trivial, $U(Q) = W_Q = \emptyset$, by Fact 7(a). Suppose $Q = Q_1^C$ for some Conserv, Ext, and Isom Q_1, and Q is non-trivial. Then Q lives on C, and $C = U(Q)$ by the theorem, so, by Fact 8, $W_Q \subseteq U(Q)$. Also, since Q lives on W_Q, it follows by Lemma 18 that $U(Q) \subseteq W_Q$. So in both cases, $U(Q) = W_Q$. □

Putting together all of the above, we obtain a final characterization of Isom decomposability:

Corollary 21
(Fin) *Q is* Isom *decomposable if and only if Q is* Ext, *W_Q exists, and Q doesn't distinguish subsets of W_Q of the same size.*

The restriction to finite universes is essential for the results in this subsection. For example, the quantifier mentioned in footnote 2, $Q =$ *infinitely many numbers*, is non-trivial and Isom decomposable, $W_Q = \emptyset$, Q doesn't live on any finite set, and for each n, $Q = (\textit{infinitely many})^{N-\{0,1,\ldots,n\}}$ (where $N = \{0, 1, 2, \ldots\}$).

4.5 Some instructive examples

Suppose $Q = Q_1^C$, where Q_1 is *not* Isom. Can we conclude anything about the Isom decomposability of Q? No: even if Q Isom decomposable, it is equal to Q_1^C for infinitely many C and infinitely many non-Isom Q_1. We give some examples to illustrate that natural quantifiers of the form Q_1^C with non-Isom Q_1 are sometimes Isom decomposable and sometimes not.

In section 2.1 we defined simple possessive quantifiers like *John's* by

$$\textit{John's}_M(A, B) \iff \emptyset \neq A \cap R_j \subseteq B$$

(where $R_j = \{b : R(j, b)\}$, and $j =$ John). This is a special case of

$$Q_2 \textit{ of John's}_M(A, B) \iff \emptyset \neq A \cap R_j \ \&\ Q_2(A \cap R_j, B)$$

Cf. sentences like

(17) John's bikes were (all) stolen.

(18) Most of Mary's friends are here.

(19) All but two of Henry's job applications failed.

Here $Q_2 =$ *all*, *most*, and *all but two*, respectively. None of the possessive type $\langle 1, 1\rangle$ quantifiers in (17) – (19) is Isom, but we still have:

Fact 22
If Q_2 is Conserv, Ext, *and* Isom, *quantifiers of the form $Q = (Q_2 \textit{ of John's})^C$ are* Isom *decomposable. Moreover* (Fin), *$W_Q = C \cap R_j$.*

Proof. Q is clearly Ext. We have

(20) $Q(B) \iff \emptyset \neq C \cap R_j \;\&\; Q_2(C \cap R_j, B)$

If $C \cap R_j = \emptyset$, then Q is trivial, hence trivially Isom decomposable, and $W_Q = \emptyset$ (Lemma 7(a)). If $C \cap R_j \neq \emptyset$, then, by (20), $Q = (Q_2)^{C \cap R_j}$, and hence is Isom decomposable by the assumption on Q_2. Also, if Fin holds, we see from Corollary 20 that $W_Q = C \cap R_j$. □

Thus, the smallest set that, say, *John's*^{books} lives on is not the set of books in the discourse universe, but the set of books owned by John (if $R = owns$); cf. Corollary 21.

What about other frozen possessive quantifiers? I will not try to answer this question in any generality here, but only give a few more illustrative examples. Consider the determiner *some student's*. This has (at least) two readings, a *universal* and an *existential* one, illustrated by

(21) Some student's tennis rackets were stolen.

(22) Some student's books were left in the classroom.

A plausible reading of (21) is that some student had all his or her tennis rackets stolen, whereas (22) can be taken to mean that some, but not necessarily all, of the student's books had been left in the room. The two readings are given by, respectively,

(23) $(\textit{some } D\textit{'s})^u(A, B) \iff \exists a \in D\, (\emptyset \neq A \cap R_a \subseteq B)$

(24) $(\textit{some } D\textit{'s})^e(A, B) \iff \exists a \in D\, (\emptyset \neq A \cap R_a \cap B)$

Fact 23
$Q = ((\textit{some } D\textit{'s})^e)^C$ is Isom *decomposable, for all D, C, and R.*

Proof. We use Theorem 15. Let

(25) $C_0 \;=\; C \cap \bigcup_{a \in D} R_a$

It is not hard to verify that

(26) Q lives on C_0.

Also, it follows from the definition of $(\textit{some } D's)^e$ that

(27) If $\emptyset \neq B \subseteq C_0$, then $Q(B)$ holds.

This entails that Q does not distinguish subsets of C_0 of the same size. Since Q is clearly Ext, it follows from Theorem 15 that Q is Isom decomposable. □

Although I will not prove it here, it is a general fact about possessive quantifiers frozen to a set C that they live on the set C_0 defined in (25) above. In particular, $((\textit{some } D\textit{'s})^u)^C$ does so, and we see that (with $D = \{j\}$) the quantifiers $(Q_2 \textit{ of John's})^C$ do so too. In this latter case we have $W_Q = C_0 = C \cap R_j$, but in general W_Q can be a proper subset of C_0. To give an example, suppose

$D = \{s_1, s_2\}$, $C = \{b_1, b_2\}$, and $R = \{(s_1, b_1), (s_2, b_1), (s_2, b_2)\}$. Then $C_0 = C$, and one easily verifies that with $Q = ((\textit{some } D\textit{'s})^u)^C$, we have $\neg Q(\emptyset)$, $Q(\{b_1\})$, $\neg Q(\{b_2\})$, and $Q(\{b_1, b_2\})$. From this one sees that

$$W_Q = \{b_1\}$$

For this particular choice of C, D, R we also have that $((\textit{some } D\textit{'s})^u)^C$ is Isom decomposable. This too is a consequence of Corollary 21:

Fact 24
(Fin) *If Q is* Ext*, and W_Q exists and has at most one element, Q is* Isom *decomposable.*

Proof. Then Q trivially doesn't distinguish subsets of W_Q of the same size! □

However, a similar example shows that a slightly different possessive quantifier need not be Isom decomposable. Consider

(28) (Exactly) two students' dorm rooms were burglarized.

One plausible interpretation is

(29) $\textit{two } D\textit{'s}(A, B) \iff |D \cap \{a\colon \emptyset \neq A \cap R_a \subseteq B\}| = 2$

Let $D = \{s_1, s_2, s_3\}$, $C = \{b_1, b_2\}$, and $R = \{(s_1, b_1), (s_2, b_1), (s_3, b_2)\}$. Again $C_0 = C$, and one sees that with $Q = (\textit{two } D\textit{'s})^C$, we have $\neg Q(\emptyset)$, $Q(\{b_1\})$, $\neg Q(\{b_2\})$, and $\neg Q(\{b_1, b_2\})$. For example, in the last case: the number of Ds such that every C they are related to by R belongs to $\{b_1, b_2\}$ is 3, not 2. But then Q lives neither on $\{b_1\}$ nor on $\{b_2\}$, although it does live on $C_0 = \{b_1, b_2\}$. So $W_Q = \{b_1, b_2\}$, and it follows that Q does distinguish some subsets of W_Q of the same cardinality (namely, $\{b_1\}$ and $\{b_2\}$). We have shown:

Fact 25
There are D, C, and R such that $(\textit{two } D\textit{'s})^C$ is not Isom *decomposable.*

We see that the Isom decomposability (or not) of frozen possessive quantifiers sometimes depends on the choice of the 'possessor' relation. For example, we can make the following observation:[4]

Fact 26
If R is a (partial) function, then, for all D and C, $Q = ((\textit{some } D\textit{'s})^u)^C$ is Isom *decomposable.*

Proof. That R is a partial function means that for all a, R_a has at most one element. From this, and the fact that Q is *monotone* ($Q(B)$ and $B \subseteq B'$ implies $Q(B')$), one verifies that, with C_0 as in (25),

(30) for $B \subseteq C_0$: $Q(B) \iff B \neq \emptyset \;\&\; \forall b \in B\, Q(\{b\})$.

[4]Thanks to Christian Bennet for the suggestion.

It follows that for all non-empty subsets B of C_0 we have $Q(B)$. So in particular, Q does not distinguish subsets of C_0 of the same size. Since Q lives on C_0 and is Ext, Q is Isom decomposable, by Theorem 15. $\square$

However, most 'possessor' relations, like real ownership, are not partial functions (one normally owns many different things).

Note finally that, although the proof of Fact 25 makes use of Corollary 21, it identifies a finite set as the smallest set the quantifier lives on. This is independent of the size of the universe; a finite counter-example to Isom decomposability is given, and this fact, in contrast with positive results of Isom decomposability using Corollary 21, like Fact 26, doesn't rely on Fin. The same holds for our final example:

Fact 27
There are D, C, and R such that $Q = ((\textit{some } D\textit{'s})^u)^C$ is not Isom *decomposable.*

Proof. Let $D = \{s_1, s_2, s_3\}$, $C = \{b_1, b_2, b_3, b_4\}$, and (draw a diagram!) $R = \{(s_1, b_1), (s_2, b_1), (s_2, b_2), (s_3, b_3), (s_3, b_4)\}$. As to subsets of $C_0 = C$ we then have:

(i) Q holds of $\{b_1\}$ and its supersets, and of $\{b_3, b_4\}$ and its supersets.

(ii) Q does not hold of: $\emptyset$, $\{b_2\}$, $\{b_3\}$, or $\{b_4\}$, nor of $\{b_2, b_3\}$ or $\{b_2, b_4\}$.

Using the monotonicity of Q, and going through the various cases, one can verify that

(30) Q lives on $\{b_1, b_3, b_4\}$,

and then that

(31) $W_Q = \{b_1, b_3, b_4\}$

So Q distinguishes subsets of W_Q of the same size (e.g. $\{b_1\}$ and $\{b_3\}$), and hence is not Isom decomposable. $\square$

Observe also that

$$((\textit{some } D\textit{'s})^u)^C = \bigvee_{a \in D} (\textit{all of } a\textit{'s})^C$$

so we have another illustration of the fact that Isom decomposability is not closed under disjunction (cf. Fact 16).

5 Applications

Noun phrases abound in most languages, as immediate constituents of other phrases: sentences, verb phrases, prepositional phrases, etc. In a compositional semantics, the semantic value of the larger phrase is determined by the values of the immediate constituents (and the 'mode of composition'). Suppose the value of the NP is of the form Q^C, but calculating the value of the larger phrase requires access to C and/or Q. Then we have at least a *prima facie* problem for compositionality, since the phrases whose values are Q and C are not *immediate* constituents of the larger phrase. Does this situation actually occur? We look at two cases.

5.1 Reciprocals

Consider the sentences[5]

(32) Most of the pirates stared at each other in surprise.

(33) Most of the boys in my class know each other.

In examples like this, *each other* can be construed as a type $\langle 1, 2\rangle$ quantifier EO, where $EO(A, R)$ says roughly that all individuals in A 'R each other'. But this can mean different things. For example, in (33), we seem to have

$$EO_1(A, R) \iff \forall a, b \in A(a \neq b \;\Rightarrow\; R(a, b))$$

(32), on the other hand, rather uses EO_2:

$$EO_2(A, R) \iff \forall a \in A \exists b \in A(a \neq b \;\&\; R(a, b))$$

(It is difficult to stare at more than one person at a time.)

The meaning of the whole sentence can then be obtained as a 'Ramsey lift' along the following scheme:

(34) $Ram_i(Q)(C, R) \iff \exists X \subseteq C(Q^C(X) \;\&\; EO_i(X, R))$

That is, if Q is a type $\langle 1, 1\rangle$ quantifier like *most* (one usually assumes that the quantifier is increasing in the right argument, like *most*, or *John's*), (34) gives a type $\langle 1, 2\rangle$ quantifier $Ram_i(Q)$ suitable for interpreting certain reciprocal sentences.

The most natural compositional analysis of sentences like (32) and (33) would seem to go like this:

[Det C] [R each other]

$Q^C \quad \lambda X\, EO(X,R)$

But to get the meaning of the sentence, according to (34), at the last step, we in general need access not only to Q^C but also to C. Thus, Theorem 19 applies, when Q^C is ISOM decomposable (like *most pirates* or *John's companions*) and only finite universes are considered. Since $C = U(Q^C) = W_{Q^C}$, this theorem guarantees the compositionality of the corresponding reciprocal constructions.[6]

[5] See Dalrymple et al. [3] for a treatment of the semantics of reciprocals. (32) is a variant of a sentence discussed there.

[6] One may note that since Q is CONSERV,

$$Ram_1(Q)(C, R) \iff \exists X(Q^C(X) \;\&\; EO_1(X, R))$$

5.2 Possessives

Next, consider

(35) Most planets' rings are made of ice.

Here too we have an NP, *most planets*, whose interpretation is of the form Q^C, and which is naturally seen as an immediate constituent of the phrase *most planets'* (which can be taken to be a determiner). But in this case, access to C doesn't seem to be enough. This is because *most* in (35) doesn't really quantify over all the planets, but only over those that have rings. Planets without rings are irrelevant to the truth value of (35). The phenomenon was called *narrowing* in Barker [1] (from which the example is taken).

Thus, we need access to $Q^{C'}$, for some $C' \subseteq C$. But, as we have seen, this is in general not possible. One cannot recover Q from Q^C, and therefore not $Q^{C'}$ either, when $C' \neq C$, even if C' is known. In Peters and Westerståhl [5] we analyze the situation and conclude that this appears to constitute a serious problem for the compositionality of the semantics for possessives.

We also note a related problem for possessive semantics. If one construes the semantics so that not Q^C but both Q and C are taken as arguments when calculating the meaning of possessive phrases, one has an additional problem besides compositionality: What to do when the NP is not quantified, as in (36) and (37)?

(36) Mary's grant applications were successful.

(37) John or Mary's grant applications were successful.

The obvious answer is: Decompose! But one problem is that such decomposition is not unique. For example, $I_m = all^{\{m\}} = (all_{\text{ei}})^{\{m\}}$, and the truth conditions vary slightly with the decomposition chosen.[7] Another issue is that phrases of the form [NP 's] are sometimes problematic. Consider

(38) (?) John, or Mary and Sue's grant applications were successful.

(39) (?) Some man or every woman's grant applications were successful.

Many speakers find (38) and (39) meaningless or at least very strange.[8] Our logical results may apply here too. For example, we note that the NPs in (38)

So in this case, access to C is in fact not needed. This is because EO_1 is defined by a universal condition. But EO_2 is not, and C cannot be similarly eliminated for $Ram_2(Q)$.

[7] On our account, if one uses $all^{\{m\}}$, (36) becomes true iff either Mary didn't make any grant applications or all of her applications were successful. With $(all_{\text{ei}})^{\{m\}}$, it is – more reasonably – true iff she made at least one grant application and all her applications were successful.

[8] Note, *first*, that we are of course disregarding readings of (39) which say that either some man was successful or every woman's grant application was successful, and, *second*, we realize that there is a tendency to read (39) as the unproblematic

(i) Some man's or every woman's grant applications were successful.

But it could still be maintained that this is just a way to charitably make sense of or 'save' the deviant (39).

and (39) are not Isom decomposable (Fact 16), in contrast with those in (36) and (37). These issues too are discussed further in [5].

In conclusion, the ubiquity of linguistic constructions involving phrases whose interpretations are type $\langle 1 \rangle$ quantifiers, decomposable or not, makes it likely that the simple logical facts about decomposition established in this note are relevant to various issues in natural language semantics.

References

[1] Chris Barker. *Possessive Descriptions.* CSLI Publications, Stanford, 1995.

[2] Jon Barwise and Robin Cooper. Generalized quantifiers and natural language. *Linguistics and Philosophy*, 4:159–219, 1981.

[3] M. Dalrymple, M. Kanazawa, Y. Kim, S. Mchombo, and S. Peters. Reciprocal expressions and the concept of reciprocity. *Linguistics & Philosophy*, 21:159–210, 1998.

[4] Stanley Peters and Dag Westerståhl. *Quantifiers in Language and Logic.* Oxford University Press, Oxford, 2006.

[5] Stanley Peters and Dag Westerståhl. The semantics of possessive constructions. In preparation.